Automatic Error Function Learning with Interpretable Compositional Networks

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Outline

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What is Constraint Programming?

What is CP?

Set of methods to model and solve combinatorial problems.

Constraint Network

A constraint network (CN) is defined by a tuple (V, D, C) such that:

$$\text{CN} = \left[\begin{array}{ll} V: & \text{Set of variables.} \\ D: & \text{Domain (set of possible values of variables).} \\ C: & \text{Set of constraints (\textit{i.e.}, predicates).} \end{array}\right.$$

Constraint Satisfaction Problem (CSP)

Given a constraint network, does a solution exist?

Example: the 3-color problem

CN for 3-color

Variables $V = \{v_1, \ldots, v_n\}$, one variable for each vertex.

Domain $D = \{0, 1, 2\}$, one value for each color.

Constraint $\neq$, one per edge.

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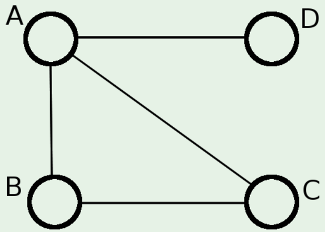

CSP formula

$(a \neq b) \wedge (a \neq c) \wedge$
$(b \neq c) \wedge (a \neq d)$

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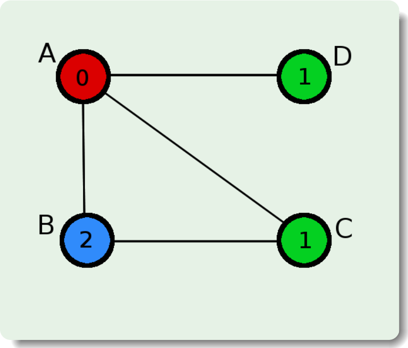

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$(a \neq b) \wedge (a \neq c) \wedge$
$(b \neq c) \wedge (a \neq d)$

A solution

$a = 0$, $b = 2$, $c = d = 1$

Intuitive definition of EFN

Error Function Networks (EFN)

$$\text{EFN} = \left[\begin{array}{ll} V: & \text{Set of variables.} \\ D: & \text{Domain (set of possible values of variables).} \\ F: & \text{Set of error functions } f_c : D^k \to \mathbb{R}^+. \end{array}\right.$$

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Intuition behind error functions

Let (x_1, x_2, x_3) be an assignment for f_c:

- If $f_c(x_1, x_2, x_3) = 0$ then (x_1, x_2, x_3) **satisfies** the constraint c.
- If $f_c(x_1, x_2, x_3)$ is small then (x_1, x_2, x_3) is **close to** satisfy c.
- If $f_c(x_1, x_2, x_3)$ is high then (x_1, x_2, x_3) is **far from** satisfying c.

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Error Function Satisfaction Problem (EFSP)

Given a error function network, does a solution exist?

Intuitive definition of EFN

Constraint representation

Error function = degree of dissatisfaction of a constraint.

For example

Consider $f_c(x, y) := |x - y|$ (representing the constraint $x = y$)

- With $x = 4$ and $y = 4$, $f_c(4, 4) = 0$
- With $x = 4$ and $y = 5$, $f_c(4, 5) = 1$
- With $x = 4$ and $y = 500$, $f_c(4, 500) = 496$

Outline

Why EFN?

Offers a landscape on assignments $\vec{x}$.

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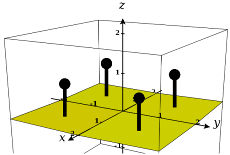
No structures

$$l(\vec{x}) = \begin{cases} 1 & \text{if } \vec{x} \text{ is a solution} \\ 0 & \text{otherwise} \end{cases}$$



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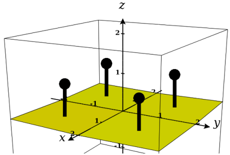

Constraint Network

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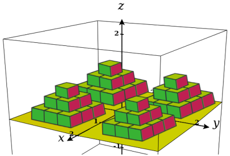

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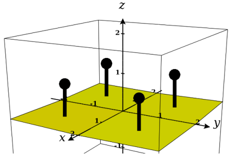

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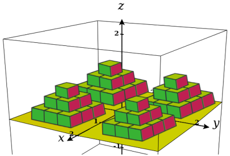
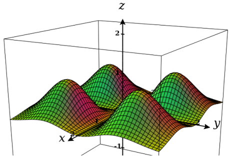


Error Function Network

$$l(\vec{x}) = \sum_{f_c \in F} f_c(\vec{x})$$



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Is $f_c(x, y) = |x - y|$ relevant for the constraint $x = y$?

- If $x = 4$ and $y = 5$, then change y to 4 or x to 5 $\Rightarrow$ 1 action.
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Learning Error Functions

Error functions seen as (non-linear) combination of elementary operations.

Goal

For each constraint, learn a good combination of elementary operations.

The user provides a CN

$$\left[\begin{array}{ll} V: & \text{Variables} \\ D: & \text{Domain} \\ C: & \text{Constraints} \end{array}\right.$$

The user gets an EFN

$$\left[\begin{array}{ll} V: & \text{Variables} \\ D: & \text{Domain} \\ \mathbf{F}: & \textbf{Error functions} \end{array}\right.$$

Learning Error Functions

Supervised learning

Learn error functions similar to the Hamming error.

Hamming error $h_c(\vec{x})$

$h_c(\vec{x})$: minimal number of values from $\vec{x}$ to change to get a solution.

Loss function of our supervised learning

Let θ_c be our model for one error function f_c.

$$\mathcal{L}(\theta_c, h_c) = \sum_{\vec{x}} |\theta_c(\vec{x}) - h_c(\vec{x})|$$

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So what is our model θ_c?

Idea based upon CPPN

Our model is a variation of Compositional Pattern-Producing Networks.

Regular neural networks

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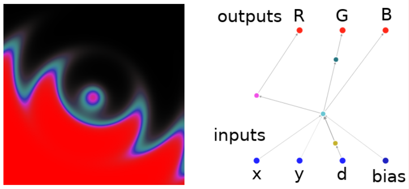
CPPN used to make 2D/3D images (source: otoro.net/neurogram/)



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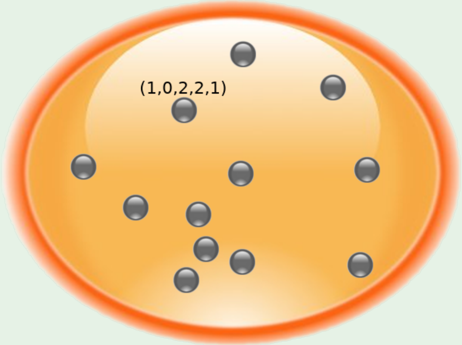
Input space of **one given constraint**



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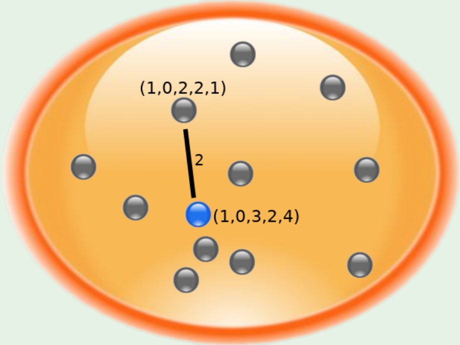
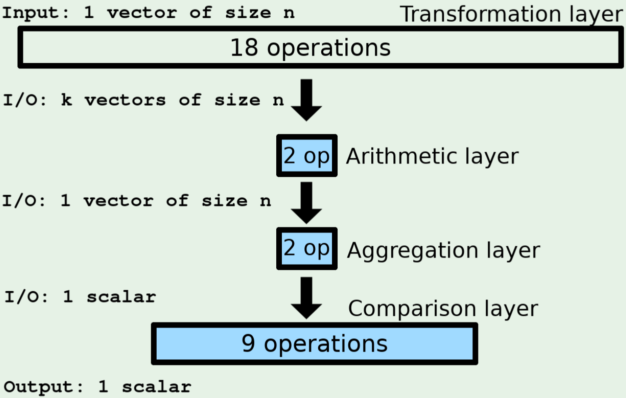


Our ICN architecture



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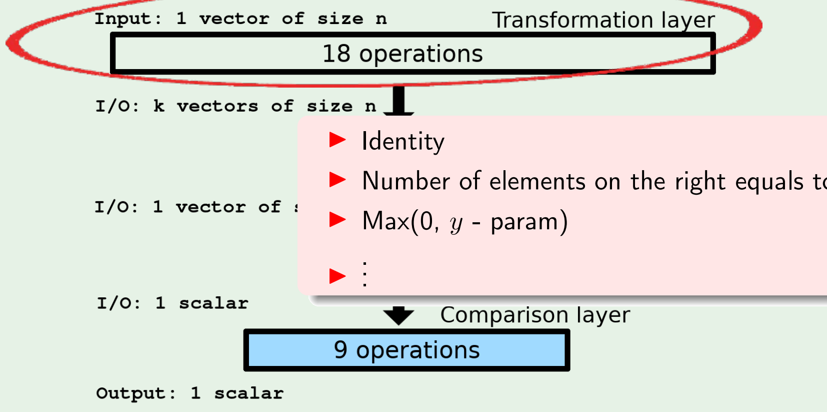

- Identity
- Number of elements on the right equals to y
- Max(0, y - param)
- $\vdots$

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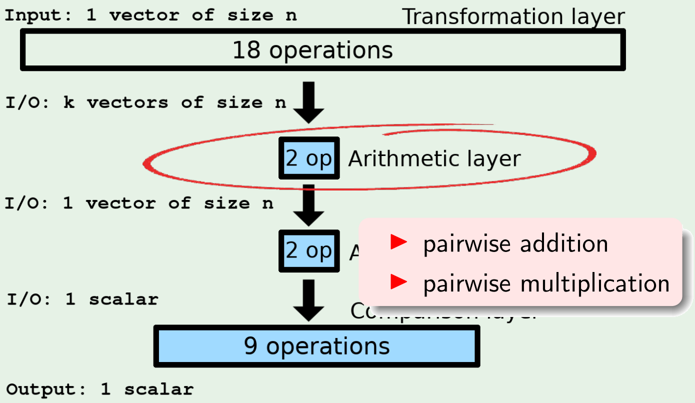

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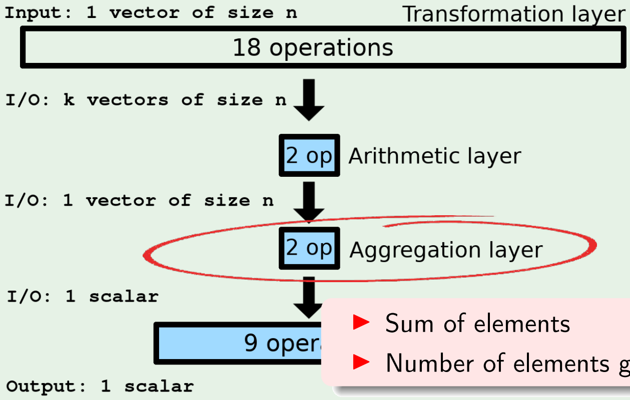

- Sum of elements
- Number of elements greater than 0

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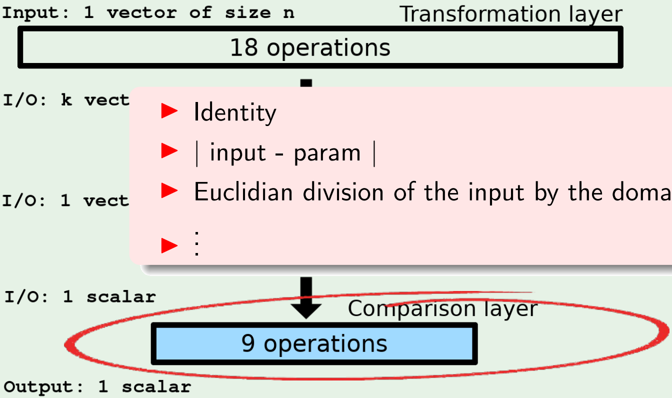

- Identity
- | input - param |
- Euclidian division of the input by the domain size
- ⋮

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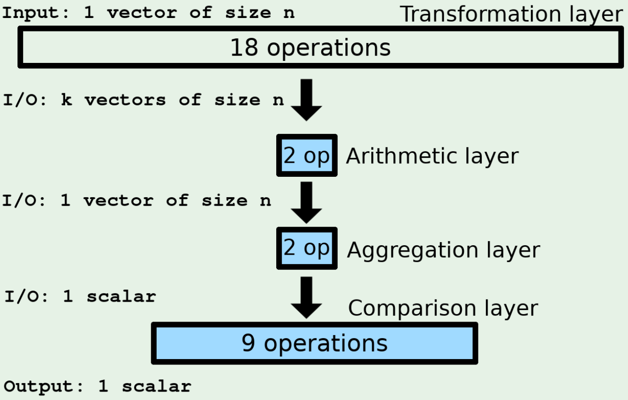

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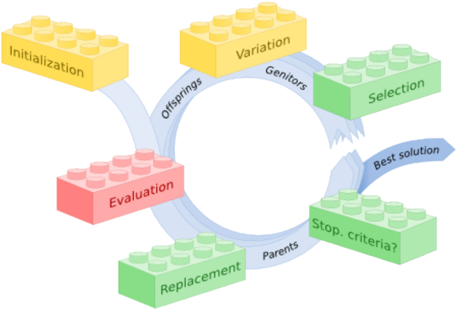
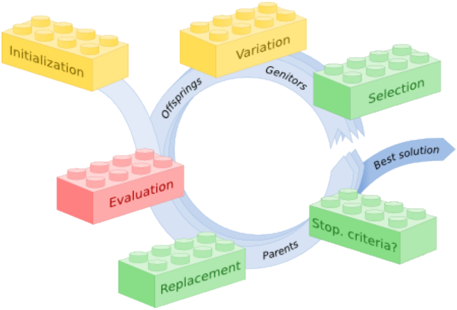


Individual modeling

Vector of 29 bits:
one bit for each operations.



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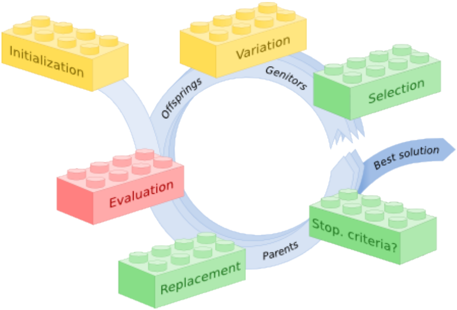

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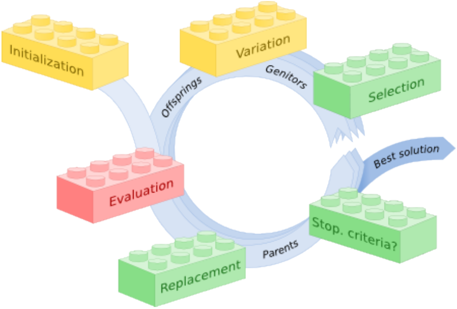

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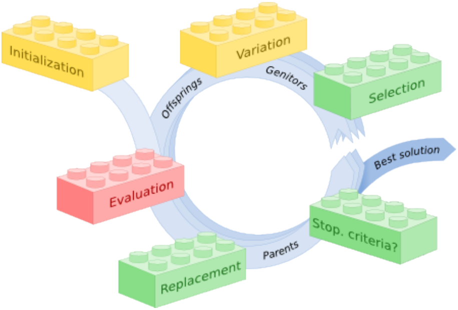

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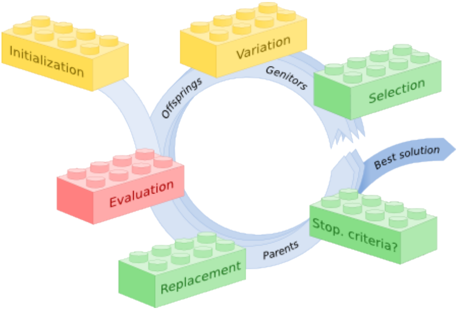
Reaching 400 generations.



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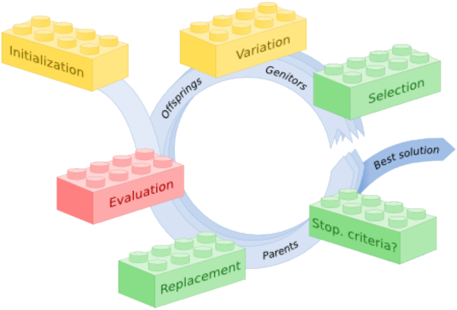

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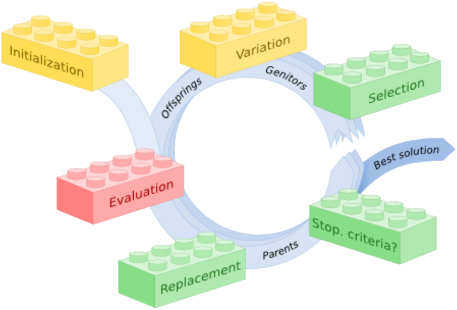

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Variation

One-point crossovers.
One-flip mutations.

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Tested constraints

5 major constraints:

- **All different**: variables must all be assigned to different values.
- **Ordered**: assignment of n variables $(x_1, \ldots, x_n)$ must be ordered, given a total order.
- **Linear sum**: equation $x_1 + x_2 + \ldots + x_n = p$ must hold.
- **No overlap**: variables represent tasks with a given length. A variable's value is its task starting time. No tasks must overlap.
- **Minimum**: the minimum value of an assignment must check a given numerical condition.

Three experimental protocols

Exp. 1: Scaling

Question: Do error functions learned over small spaces scale?

- Learn error functions over small spaces ($\simeq 500$ assignments),
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Exp. 3: Solving Sudoku with learned error functions

Question: Can a learned error function be used to solve an actual problem?

- Solve Sudoku with and without error functions.

Experimental result 1: Scaling

Constraints	median	mean	most freq.
all different	0	0.03	0 (97)
ordered	0.08	0.08	0.08 (100)
linear sum	0.01	0.05	0.01 (74)
no overlap	0.14	0.19	0.11 (50)
minimum	0	0.04	0 (88)

Table: Training error over small spaces (500 assignments).

all_diff	ord	lin_sum	no_ol	min
0	1.27	0.03	2.68	0

Table: Mean test error over 20,000 assignments in huge spaces.

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Test spaces of size 10^{200}, but...

- Not easy to compute the Hamming error for Ordered and NoOverlap.
- Estimation of their Hamming error over spaces of size $\simeq 10^{15}$.

Experimental result 2: Incomplete spaces

Constraints	median	mean	most freq.
all different	0.44	0.44	0.44 (99)
ordered	0.44	0.46	0.44 (66)
linear sum	2.03	1.70	0.85 (37)
no overlap	2.33	2.39	2.29 (48)
minimum	0.59	0.59	0.59 (78)

Table: Training error over incomplete large spaces ($\simeq 50.000$ assignments).

all_diff	ord	lin_sum	no_ol	min
0	1.80	0.03	2.02	0

Table: Mean test error over 20,000 assignments in huge spaces.

Experimental result 3: Sudoku

Error Function	mean	median	std dev	min	max
no error functions	1044	764	727	250	3546
learned	383	331	268	57	1812
hard-coded	175	145	107	46	662
hand-crafted	149	125	107	26	608

Table: Run-times in milliseconds over 100 runs to solve Sudoku.

Rows in this table

1. No error functions (pure CN),
2. The most frequently learned error function for *All different* in Experiment 1 run through the ICN,
3. The same function but hard-coded in C++,
4. A hand-crafted error function (Petit et. al 2001).

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Conclusion

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- Interpretable model.
- Scale!
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- Use it fully automatically or as a decision support system.

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Perspectives

- Need more diverse and expressive operations for very combinatorial constraints (Ordered, No overlap).
- Reinforcement learning to find error functions adapted to the solver.

Paper and source code

Get the paper!

arxiv.org/abs/2002.09811

Get the code!

github.com/richoux/LearningCostFunctions

Questions?

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